Chapter 5 At-a-Glance

Chapter 5 is the second of four subject-specific chapters that will address ways in which you can help your ELLs meet the standards that are an increasing factor in measuring the success of teaching and learning. Middle and high school teachers, as you know, are primarily focused on their subject areas, and these chapters are designed to supplement the more general information presented in Chapter 3. Also, for a much more detailed discussion of assessment strategies and resources, please refer to Chapter 8, "Assessment and Evaluation: How Can We Be Fair and Demanding?"

Some highlights of Chapter 5 include:

- Emphasizing problem solving in authentic contexts
- Encouraging development of critical thinking skills
- Teaching the language of mathematics
- Varying instructional methods
- Assessing ELLs within a mathematics program

The bulk of this chapter is adapted from "Preparing Secondary Education Teachers to Work with English Language Learners: Mathematics," by Kris Anstrom, with contributions from Patricia DiCerbo (1998). The project was funded by the U.S. Department of Education, Office of Bilingual Education and Minority Language Affairs. The paper in its entirety is available at the National Clearinghouse for Bilingual Education at the George Washington University website at www.ncbe.gwu.ed.

Emergent English speakers can reason at a much deeper level than they have the vocabulary to express.

—Kris Anstrom

CHAPTER 5:
Making Math Comprehensible to the English Language Learner

Introduction

A central focus in the move toward standards-based learning has been the question of how to ensure equal access to a quality education, with particular attention paid to the special needs of linguistically diverse English language learners (ELLs). In comparison to their English proficient peers, students who are in the process of learning English face a more difficult task in meeting a common set of learning standards. Not only do these students have to focus on the cognitive demands of a given class or assignment—learning new subject matter, procedures, and tasks—they have to do so while learning new vocabulary, linguistic structures, and academic discourse. Moreover, ELL students at the secondary level may have significant gaps in their prior education that influences their ability to meet the cognitive demands of the high school curriculum (McKeon, 1994).

A misconception exists among educators that since mathematics uses symbols, it is culture-free and ideal for facilitating the transition of recent immigrant students into English instruction. This concept is true only of mathematical computations and cannot be applied to a context-based curriculum that stresses understanding and communication (Lang, 1995). Mathematics achievement statistics show that Hispanics, a group with many emergent English speakers, have not excelled in mathematics and are significantly underrepresented in all scientific and engineering careers (Garrison, 1997).

Emphasize Problem Solving in Authentic Contexts

Efforts to reform the teaching of mathematics were given a push in 1989 with the release of Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics or NCTM). This and other reform documents criticized the mathematics curriculum and its instructional methodology. Standards developers advocated moving away from the traditional focus on acquisition of facts, technical skill, and textbook-based instruction to a curriculum of hands-on activities and intellectually challenging problems. According to the report, traditional teaching's emphasis on practice in manipulating expressions and practicing algorithms as a precursor to solving problems ignores the fact that knowledge often emerges from the problems. This suggests that instead of the expectation that skill in computation should precede word problems, experience with problems helps develop the ability to compute. Thus, present strategies for teaching may need to be reversed; knowledge often should emerge from experience with problems. Furthermore, students need to experience genuine problems regularly. A genuine problem is a situation in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed (National Council of Teachers of Mathematics, 1989).

The notion that students will develop their mathematics abilities to a higher degree when motivated by authentic problems is supported by research (McLaughlin & McLeod, 1996) and has led to changes in instructional practices within some school districts. Below, two classroom vignettes demonstrate how problem solving in authentic contexts has been used to improve learning for students from diverse cultural and linguistic backgrounds.

Vignette I.

Applying Mathematical Skills to Solve Problems

In a seventh-grade classroom in Salinas, California, Latino students are huddled over a model of a bridge that they have constructed. They are trying to determine the proportions needed to build a slightly different bridge—one that is three-and-a-half times larger.

By focusing instruction on such themes as architecture (bridges), astronomy (space), and statistics (baseball), mathematics is taught in highly contextualized situations where the focus is on the acquisition of conceptual knowledge, problem solving, and application of mathematical skills to concrete problems (McLaughlin & McLeod, 1996).

Vignette II.

Developing Creative Mathematical Problem Solving

Ms. Simis' eighth-grade math class includes 15 LEP (limited English proficient) students. They speak a number of primary languages and have varying degrees of oral English proficiency, but have sufficient English reading and writing skills to participate in an all-English environment.

Ms. Simis conducts the class in English. The next lesson is about spatial math. At the start of the lesson, the teacher tells the students that she once found this aspect of mathematics to be difficult. With patience and persistence, she says, the students will understand it just as she did. Ms. Simis asks students to design three-dimensional buildings using Legos, following specific constraints: preserve the right and front view but extend the building.

As she introduces the lesson, she asks a student to restate her constraints and instructions; after the second attempt he does so. She then suggests that student "experts" in each group assist other students. The experts are not necessarily the most accomplished; Ms. Simis selects students who have struggled to learn something, so they can help others who are struggling.

The students use Lego blocks to model the building and then draw the structure from all angles on special paper. As they extend the buildings in new ways, Ms. Simis calls these innovations to the attention of the class. When one group says, "We're finished," she challenges, "Now solve it another way" (Berman et al., 1995).

Encourage Development of Critical Thinking Skills

The shift in focus on the part of the mathematics community parallels current thinking about how to best educate ELLs. Padron (1993), for example, argues that the traditional notion of educating disadvantaged students in basic skills before exposing them to more challenging academic material has led to what they term "learned helplessness" or limited mastery of cognitive skills. For ELL students, a basic skills mastery approach can result in an inability to solve problems, to reason effectively, and to develop other higher-order thinking skills. Effectively teaching mathematics content to ELLs requires instructional settings and situations—such as those described in the previous vignettes—where students are engaged in solving interesting, real-life problems that encourage critical thinking along with basic skills development and practice.

Emergent English speakers can reason at a much deeper level than they have the vocabulary to express. Teachers, therefore, must work with students to help them develop and communicate their mathematical reasoning. Although teachers should work to diminish the complexity of the language used in a lesson, they should strive to maintain the complexity of thought.

Practical suggestions for math teachers include:

- Striving to rely less on verbal explanations and more on the use of diagrams, paper models to manipulate, charts, graphs, and real objects (Garrison, 1997).
- Evaluating the responses of ELLs carefully for evidence of clear reasoning.
- Setting high expectations for all students and not letting simple sentences, grammar, or spelling errors hide evidence of good thinking.
- Regarding any comments on the writing and oral presentations of ELLs, teachers should focus primarily on the quality of the reasoning and only secondarily on language. This emphasis encourages students to keep developing their mathematical-reasoning abilities while they are learning to master English.

Teach the Language of Mathematics

Command of mathematical language plays an important role in the development of mathematical ability.

Mathematics courses can provide the necessary experiences for ELL students to acquire higher-order thinking skills and mathematical competencies while also improving their communicative abilities in English. For such learning to occur, though, students need ample opportunities to hear math language and to speak and write mathematically. NCTM guidelines explicitly address this issue by directing teachers to orchestrate problem solving and other classroom discourse in a manner that encourages mathematical literacy (Buchanan & Helman, 1993). The guidelines recommend that teachers pose questions and design tasks that engage students' thinking and ask students to clarify and justify ideas orally and in writing (National Council of Teachers of Mathematics, 1989).

NCTM guidelines and the research on language learning point out that mathematics discourse and syntactical structures have a number of features that make it difficult for ELLs to gauge meaning, such as the use of symbols and technical language, and the lack of redundancy or paraphrase to assist in understanding. Statements and questions are often written in the passive (for example, ten [is] divided by two). The language of mathematics also includes vocabulary specific to the field, such as equation or algebraic, as well as everyday vocabulary that has different meanings when used in mathematical contexts, such as positive and negative, table and irrational. Strings of words, such as "measure of central tendency" and "square root" create complex phrases with specific meanings. Mathematics operations can often be signaled by more than one word or phrase; for example, "add," "plus," "combine," "sum," and "increased by all" indicate addition (Corasaniti Dale & Cuevas, 1992).

It is important to find out whether your students who have attended school in another country use the same processes for computation and measurement. For example, in the U.S., the units of measurement (pounds, feet, inches) are unfamiliar to students who have learned the metric system. In addition, U.S. schools devote extensive practice to the use of fractions, in part because of our system of measurement. U.S. teachers often refer to "half a foot" while international teachers may refer to "five millimeters" rather than "half a centimeter."

Students who are learning English must struggle with these many discourse rules and anomalies. Mathematics teachers who work with ELL students need to employ a dual approach, incorporating instruction on the mathematical language related to the particular concepts being taught along with the concepts themselves (Corasaniti Dale & Cuevas, 1992).

Create Language-Supportive Classrooms

Journal writing offers ELLs the opportunity to practice and develop their emerging mathematics discourse skills.

Writing Word Problems

Orchestrating classroom discourse with ELL students requires more than knowledge of the intricacies of mathematical language; it demands that teachers create classroom environments and instructional situations that support and promote students' linguistic and conceptual development. For example, in working from concrete problem-solving situations to more abstract context-reduced ones, students can begin manipulating mathematical language by writing their own word problems drawn from their mathematical experiences and sharing them with other students.

Using Journals

Asking students to use journals to explore different strategies for solving mathematical problems and to shape and refine their ideas is another useful strategy. Journal writing in mathematics classrooms allows students who may be too shy or intimidated to orally communicate their ideas to "speak" freely without concern for grammar or style. In their journals, students can summarize and relate ideas, clarify concepts, and review topics. They can describe strategies, accomplishments, or frustrations, and express positive or negative emotions (Bagley

& Gallenberger, 1992). Journals can also provide a forum in which students can elaborate on their thinking and problem-solving strategies. Writing in math journals allows students the opportunity to organize and synthesize their mathematical reasoning and to practice the terms introduced in the lesson.

The following ideas for mathematical journal writing, though not specifically intended for ELL students, would be productive with these students. In some instances, native language use could be encouraged; in others, students could be asked to write in English to practice working with mathematical vocabulary and linguistic structures such as those specified in the previous section.

Prompts for Mathematical Journal Writing

1. Construct a word problem about [this] picture that can be solved mathematically. Share your problem with a partner and solve it.
2. What is the most important idea you've learned in algebra this week, and why?
3. Write a paragraph containing as many of these words as possible:_____
4. List some things you must remember when answering this type of question or doing this type of problem. (Bagley and Gallenberger, 1992)

The unit on mathematical probability described next—though, again, not specifically designed for ELL students—incorporates practices found to be effective with second language learners. By integrating reading and discussion with mathematics content, this type of unit supports the development of academic language skills and encourages greater depth in students' understanding of the topic.

Integrating Reading into a Unit on Probability

1. Students are given a written survey in which they are asked to make guesses about the probability of certain events. Through discussion, the teacher elicits students' understandings of probability and encourages questions to guide them in further exploration of the topic.
2. Students then examine the historical events that led to the invention of probability by forming pairs to read selections on probability. In pairs, students take turns reading and then stopping to pose questions and discuss the ideas presented. This "say something" strategy promotes social interaction between students, which supports their efforts to work out a meaning for the text. It also encourages them to take ownership for their reading/learning experiences and promotes an inquiry orientation to learning. Moreover, such a format works well for ELL students who may have difficulty reading textual materials on their own. By working with a partner, they are given extra support for developing both reading skills and their knowledge base on probability.
3. In follow-up exercises, the teacher asks students to put the historical information in their own words, make connections to the present, or discuss what piqued their interest while reading. By focusing on gaming in history, students can begin discussing various games in which probability plays an important role. ELL students who may have experience with different games can share these with the class at this point.
4. As the teacher begins introducing the technical aspects of probability, students review newspapers and magazines for everyday uses of probability and record these instances on note cards that can then form the basis for a discussion on how probability is interpreted in everyday usage. (Siegel & Borasi, 1992)

Connect Mathematics to Students' Background and Experiences

NCTM guidelines include a recommendation for teachers to utilize the cultural and educational background knowledge of their students as a way to help them learn mathematics and make connections to other academic fields (Buchanan & Helman, 1993). Cognitively Guided Instruction (CGI) is an approach to mathematics teaching that supports the NCTM standards by encouraging teachers to use the prior knowledge and developing mathematical thought processes of their ELLs in structuring lessons and curricula. CGI operates from the following premises:

1. Teachers must know how their students mentally organize mathematical content.
2. Instruction should focus on problem solving.
3. Teachers should determine what their students are thinking about the mathematical content studied.
4. Teachers should design instruction based on their students' thinking (Secada, 1992).

A related focus of the NCTM standards is to connect instruction to students' real-life experiences in a way that makes learning meaningful. Secondary-level students who participate in sports can learn to calculate their batting averages or race times; those who have after-school jobs can use their pay stubs to figure the percentages of their various withholding categories. Making learning relevant by connecting what happens in the classroom with the outside world has been shown to contribute to the academic achievement of ELLs and other less advantaged students (Buchanan & Helman, 1993). Such students need to believe that schoolwork makes sense for their current and long-term welfare; for this reason, classroom tasks that are intrinsically interesting or that directly relate to their interests and identity have the best chance of success (McPartland & Braddock, 1993).

Secondary teachers have found that students educated in Mexico check division answers using a procedure that is different from the one used by U.S. students. This can provide an opportunity for all students to engage in a lively discussion about how and why the Mexican procedure works and helps to bolster the esteem of bicultural students (Garrison, 1997).

Vary Instructional Methods

ELLs learn best when instructional methods and approaches match their individual abilities and learning styles.

A final caveat from NCTM advises teachers to provide students with opportunities to learn in different ways, through individual, small group, and whole class work (Buchanan & Helman, 1993). Research on effective instruction for ELL students (August & Pease-Alvarez, 1996) similarly emphasizes the importance of using a variety of methods tailored to students' needs, including direct instruction, guided discovery, cooperative learning, and computer-assisted learning. Which instructional methods are selected depends on lesson goals and objectives, learner characteristics, level(s) of English language proficiency, and available resources.

By using multiple approaches and considering individual learning styles and preferences, teachers can meet the needs of a wider variety of students (Reyhner & Davison, 1993; August & Pease-Alvarez, 1996). For example, traditional mathematics texts and materials tend to present abstract theories and verbal information. Studies of American Indian students, though, indicate that they perform better academically when taught through visual and tactile modes of learning. Developing a multi-sensory, activity-centered mathematics curriculum is one of the ways to address these types of learning style differences and support higher achievement in mathematics (Reyhner & Davison, 1993).

Sample Lesson

Mr. Smith teaches a high school math class that includes some limited-English-proficient students. As the lesson on the interior angles of a polygon begins, he notices that Maria and several other students do not seem to understand the concept of a polygon. To make sure the language foundation is in place, Mr. Smith distributes several geoboards.

Mr. Smith models and asks the students to make a shape on their geoboards. He then separates the students into two groups. He gives one group a card with the word "polygon," and the other group a card with the word "nonpolygon." Pointing to the two different groups of geoboards, he asks students to work with a partner to write what they think a polygon is. As they share their ideas, Mr. Smith writes attributes of a polygon—closed shape, no curves—on the board. He knows that this concrete, hands-on activity promotes mathematical communication. To check for understanding, the students are directed to create a polygon on their geoboards with a rubber band. As Maria holds up her polygon, Mr. Smith writes "square" on the board and pronounces the word for the class. He has modeled both math language and content for the students. Now that the students understand the concept of a polygon, Mr. Smith can go on to cement the connection between geometric vocabulary and the corresponding math content. The confusing concept of interior angles will now be easier for students like Maria to grasp.

Instructional Strategies for Math Teachers

1. Teach all the comparison words necessary to understanding and to interpreting quantitative relationships: more, less, most, least, larger, greater than, equal, half as much, twice as many.

2. Help students prepare a card file of number words. Write the word on one side and the number symbol on the reverse. Add to the file as new number words are learned.

3. Rewrite story problems in simpler English. Use short sentences, pictures, known symbols, and other illustrations that give meaning.

4. Limit the number of problems to be worked. Require a few problems representative of an essential mathematical concept.

5. Check a student's reading skills in English through informal appraisals. Ask the student to read the problem to you. Question the facts given and needed for a solution.

6. Give the special mathematical meanings for words commonly used in English, e.g. point, base, intersection, lateral.

7. Have students prepare their own glossaries of mathematical terms, including words specific to the field and words having special meanings in mathematics. Never assume understanding of unfamiliar words.

8. Encourage the use of diagrams and drawings as aids to identifying concepts and to seeing relationships.

9. Give a problem requiring several steps to solve. Have students number each step in its proper sequence. Provide practice in ordering the steps only; don't ask for an answer or solution; just for the information given, the facts needed, and the process. Ask for appropriate labels for each step.

10. Teachers should encourage beginning English speakers to write answers to complex problems in their primary language, even when the teacher does not understand it. The student, a bilingual peer, or a bilingual teacher/aide can translate it later. This strategy allows ELLs to concentrate on one skill at a time—first on their mathematical thinking, then on their English. By dividing the task into a two-step process, the student can develop both mathematical reasoning and English proficiency. As students become more fluent in English, this two-step process will no longer be needed.

Excerpted from Teaching Reading to Non-English Speakers *by E. Thonis*

ELL Assessment within a Mathematics Program

Critical to the development of a student-centered and intellectually challenging curriculum for English language learners is the implementation of an authentic and meaningful assessment plan, one that

- Has a specific and clear purpose
- Incorporates student educational experiences, parents' literacy, and other student background information
- Assesses content knowledge and abilities in English and the native language
- Includes assessment of content knowledge and language proficiency
- Uses a diversity of measures, including portfolios, observations, anecdotal records, interviews, checklists, and criterion-referenced tests (August & Pease-Alvarez, 1996)

Many of the attributes listed above are reflected in the performance assessment criteria developed by local schools and districts and described in the *Guide to Performance Assessment for Linguistically Diverse Students* (Navarrete & Gustkee, 1996). The *Guide* suggests

- Using alternative assessment procedures, such as teachers' observations and students' self-appraisals
- Designing alternative assessment tasks, including exhibits, dramatic renditions, interviews, and writing samples

Alternative assessment, because it requires students to perform authentic academic tasks similar to those originally used to teach the material, is considered a valid means of measuring student achievement. To assess mathematics proficiency, for example, students may be asked to develop a series of graphs based on student characteristics, to run a school store, or pretend to play the stock market. One of the appeals of alternative assessment is that it is continuous, allowing the teacher to track student progress toward meeting instructional objectives throughout the school year. Typically, student responses to alternative assessment tasks are organized in a portfolio designed to meet their individual needs and interests (Chamot, 1993).

The *Guide* (Navarrete & Gustkee, 1996) recommends a number of other techniques for improving assessment of ELLs in content area settings, such as the following:

- Incorporating familiar classroom material as a stimulus to assessment tasks (quotations, charts, graphics, cartoons, and works of art)
- Including questions for small group discussion and individual writing
- Mirroring learning processes with which students are familiar, such as the writing process and reading conferencing activities
- Allowing extra time to complete or respond to assessment tasks
- Designing administration procedures to match classroom instructional practices, simplifying directions in English, and/or paraphrasing in the student's native language
- Permitting students to use dictionaries or word lists

Implementing assessment approaches that accommodate students' varied learning styles and backgrounds gives assessment greater validity and usefulness (Farr & Trumbull, 1997).

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